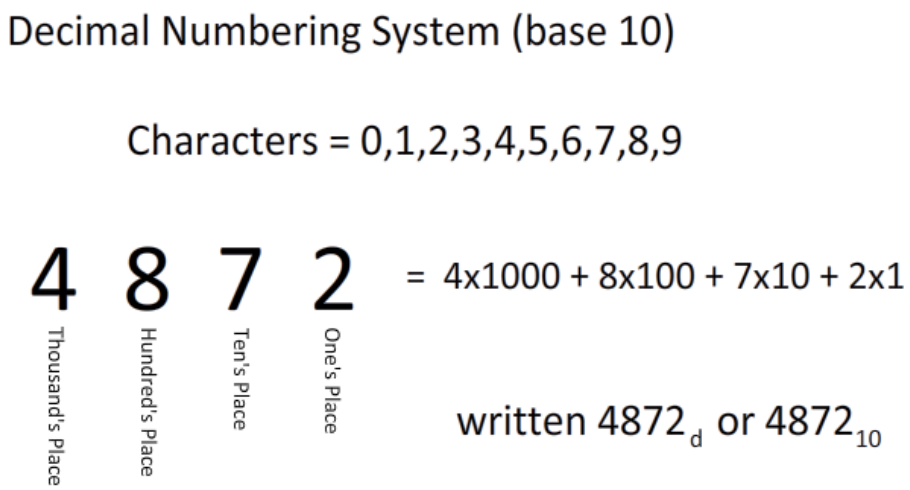
Math Circle

Number Bases



What is a Number Base:

A number base tells us how many digits we can use and what place values each digit has.

- In base 10 (our normal system), we use digits 0–9 and place values like 1s, 10s, 100s (powers of 10).
- In base 2, we only use 0 and 1, and place values are 1, 2, 4, 8, 16... (powers of 2).
- In base 8, we use digits 0–7, and place values are 1, 8, 64, 512... (powers of 8).

To convert from base 10 to another base, we divide by the base repeatedly and track remainders.

Number Bases are like counting with your fingers, the base number is the amount of fingers that you have.

Base Definition:

In a positional numeral system, the base is the number of unique digits, including the digit zero, used to represent numbers

Warmup:

What is the value of the 3 in 324?

Write 573 as a sum of place values.

What are the first five powers of 2?

Practice Problem:
Convert 100 to base 2

Explanation:

Use repeated division by 2:

- 100 ÷ 2 = 50 R0
- 50 ÷ 2 = 25 R0
- 25 ÷ 2 = 12 R1
- 12 ÷ 2 = 6 R0
- 6 ÷ 2 = 3 R0
- 3 ÷ 2 = 1 R1
- 1 ÷ 2 = 0 R1

Now write the remainders bottom to top:

1100100 base 2

Practice Problem 2:
Convert 135 to base 8

Explanation

Use repeated division by 8:

- 135 ÷ 8 = 16 R7
- 16 ÷ 8 = 2 R0
- 2 ÷ 8 = 0 R2

Write the remainders bottom to top:

207 base 8

Converting from a different base to base 10:

To convert a number from another base to base 10, multiply each digit by its place value (which is a power of the base), then add them all together.

Practice Problem 3:

Convert 203 base 4 to base 10:

Solution:

$3 * 4^0 = 3$

$0 * 4^1 = 0$

$2 * 4^2 = 32$

203 base 4 -> 35 base 10

Try by yourself problems:

1. Convert 45 to base 2
2. Convert 75 to base 5
3. Convert 60 to base 3
4. Convert 1011 base 2 to base 10
5. What's the base 10 value of 312?
6. Convert 52 to base 2
7. Convert 98 to base 4
8. Convert 41 to base 3
9. Convert 1101 base 2 to base 10
10. What's the base 10 value of 213 base 4?

Solutions:

1. 101101
2. 300
3. 2020
4. 11
5. 82
6. 110100
7. 1212
8. 1112
9. 13
10. 39