

Math Circle

Arithmetic Series

What is an Arithmetic Series?

An arithmetic series is the sum of the terms of an arithmetic progression (AP). An AP is a sequence of numbers where the difference between consecutive terms is constant. This constant difference is called the common difference, denoted by d .

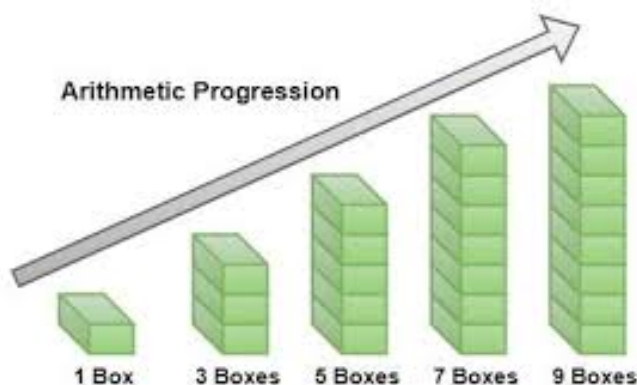
Example of an AP: 3, 7, 11, 15, ... Here, the first term $a_1 = 3$ and the common difference $d = 4$.

The formula for the n -th term of an AP is: $a_n = a_1 + (n-1)d$

The formula for the sum of the first n terms of an arithmetic series is: $S_n = (n/2)(a_1 + a_n)$ or $S_n = (n/2)(2a_1 + (n-1)d)$

Warmup

1. What is the 10th term of the sequence 2, 5, 8, ...?
2. What is the sum of the first 5 terms of the sequence 1, 3, 5, ...?
3. Find the sum of all integers from 1 to 100.



Practice Problem 1

Find the sum of the first 20 terms of the arithmetic series 5, 8, 11, ...

Explanation:

The first term $a_1 = 5$ and the common difference $d = 8 - 5 = 3$. We want to find S_{20} . First, find the 20th term, a_{20} : $a_{20} = a_1 + (20-1)d = 5 + 19(3) = 5 + 57 = 62$.

Now, use the sum formula: $S_{20} = (20/2)(a_1 + a_{20}) = 10(5 + 62) = 10(67) = 670$.

Answer: 670

Practice Problem 2

The first term of an arithmetic series is 12 and the sum of the first 8 terms is 100. Find the common difference.

Explanation: We are given $a_1 = 12$ and $S_8 = 100$. We want to find d

. Use the sum formula with $n=8$: $S_8 = (8/2)(2a_1 + (8-1)d)$ $100 = 4(2(12) + 7d)$ $100 = 4(24 + 7d)$
 $25 = 24 + 7d$ $1 = 7d$ $d = 1/7$

Answer: $1/7$

Problems:

1. Find the 30th term of the arithmetic sequence 10, 15, 20, ...
 2. Find the sum of the first 15 terms of the series 2, 6, 10, ...
 3. An auditorium has 20 rows. The first row has 12 seats, the second row has 15 seats, and so on. How many seats are in the auditorium in total?
 4. If the 5th term of an AP is 21 and the 12th term is 56, find the first term.
 5. The sum of the terms of an AP is 390. The first term is 6 and the last term is 69. Find the number of terms.
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Solutions

1. 155
2. 450
3. 810
4. 1
5. 10.4 (If the sum was 375, n would be 10)

Geometric Series

What is a Geometric Series?

A geometric series is the sum of the terms of a geometric progression (GP). A GP is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio, denoted by r .

Example of a GP: 2, 6, 18, 54, ... Here, the first term $a_1 = 2$ and the common ratio $r = 6/2 = 3$.

The formula for the n -th term of a GP is: $a_n = a_1 r^{n-1}$

The formula for the sum of the first n terms of a geometric series is: $S_n = (a_1(1-r^n))/(1-r)$

The formula for the sum of an infinite geometric series (when $|r| < 1$) is: $S_\infty = a_1/(1-r)$

Warmup

1. What is the 5th term of the sequence 3, 6, 12, ...?
2. What is the sum of the first 4 terms of the sequence 1, 2, 4, ...?
3. Find the sum of the infinite series $1 + 1/2 + 1/4 + \dots$

$$a_n = a_1 (r)^{n-1}$$

a_n = n th term

a_1 = first term

r = common ratio

n = term position

Practice Problem 1

Find the sum of the first 6 terms of the geometric series 4, 12, 36, ...

Explanation: The first term $a_1 = 4$ and the common ratio $r = 12/4 = 3$.

We want to find S_6 . Use the sum formula:

$$S_6 = (a_1(1-r^6))/(1-r) = (4(1-3^6))/(1-3) = (4(1-729))/(-2) = -2(-728) = 1456.$$

Answer: 1456

Practice Problem 2

The first term of a geometric series is 5 and the common ratio is 2. If the last term is 320, find the number of terms and the sum of the series.

Explanation: We are given $a_1 = 5$, $r = 2$, $a_n = 320$. We want to find n and S_n .

First, find n using the n -th term formula: $a_n = a_1 r^{n-1} \rightarrow 320 = 5(2)^{n-1}$
 $64 = 2^{n-1} \rightarrow 2^6 = 2^{n-1} \rightarrow 6 = n-1 \rightarrow n = 7$.

Now, find the sum of the 7 terms: $S_7 = (5(1-2^7))/(1-2) = (5(1-128))/(-1) = 5(127) = 635$.

Answer: $n=7$ and $S_7 = 635$

Problems:

1. Find the 7th term of the geometric sequence 64, 32, 16, ...
2. Find the sum of the first 5 terms of the geometric series 3, -6, 12, ...
3. A ball is dropped from a height of 10 meters. On each bounce, it reaches 80% of its previous height. What is the total vertical distance the ball travels before it stops?
4. The third term of a GP is 36 and the sixth term is 972. Find the common ratio and the first term.
5. A geometric series has a common ratio of $1/2$ and its sum to infinity is 10. Find the first term.

Solutions

1. 1
2. 33
3. 90 meters
4. $r=3$, $a_1=4$
5. 5