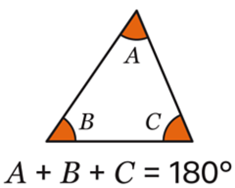
Math Circle 9/13

1 - Interior Angles

First, let's go over angles in different kinds of shapes.
Depending on the number of sides in the polygon, the sum of interior angles will differ.

For example, the sum of interior angles in a triangle is 180.
This means that all of the angles in a triangle will always add up to 180.



Another example,
We know that the sum of interior angles in a rectangle is 360.
This is because in a rectangle, all of the angles are right angles and 4 x 90 = 360
**Note: this holds true for all other quadrilaterals
So, how do we really find the sum of interior angles in a polygon?
Formula:
Sum of interior angles in a polygon with n sides = 180(n-2)

Using this formula, we can find the sum of interior angles in every polygon. Try calculating a few!
3 - sided shape: 180
4 - sided shape: 360
5 - sided shape: 540
6 - sided shape: 720
7 - sided shape: 900
8 - sided shape: 1080
9 - sided shape: 1260
10 - sided shape: 1440

Now we know how to find the sum of interior angles in a polygon.
But, why do we need to know this? Why is this important?
Well, we can use this information to find unknown angles.
Example Problems:

1. We have a pentagon ABCDE. If angle A = 112, B = 126, C = 85, and D = 65, what is the measure of angle E?
   Ans: 152

The total sum of interior angles in a pentagon is 180(5-2) = 180 x 3 = 540.
Since we know all of the other angles, we can subtract then from 540 to find the last unknown.
540 - 85 - 65 - 112 - 126 = 152
So, the measure of angle E is 152.

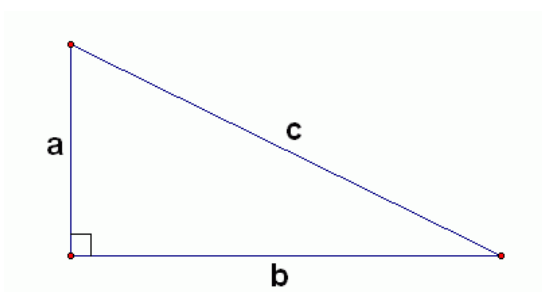
2. There is a hexagon with interior angles: 2x+1, 4x-8, x+2, x+6, 3x-5, x+4. What is the value of x?
   Ans: 60
   We know that the sum of interior angles in a hexagon is 720. Since we are given the angles in terms of x, we can add theme together to find
   2x+1+4x-8+x+2+x+6+3x-5+x+4 = 12x
   Since we know this is the sum, we set it equal to 720.
   12x = 720
   x = 60
3. In an isosceles triangle, angle B is congruent to angle C. The measure of angle A = 50. What is the measure of angle B?
   Ans: 65
   We know that the measures of angle B and C must add up to 180 - 50 = 130. Because they are also equal to each other, we just divide 130 by 2 to get 65 degrees.

2 - Pythagorean Theorem

One kind of polygon that is commonly used in math problems is a triangle.
Certain triangles, such as right triangles, have special properties.
A right triangle is a triangle with one right angle which is 90 degrees.

- Why can't we have two right angles in a triangle?
  - Look at the sum of interior angles for a triangle - if a triangle has two right angles then the last angle would have to be 180-90-90=0 - this doesn't make any sense!

The Pythagorean Theorem demonstrates a relationship between the sides of a right triangle and allows you to find the lengths if one is unknown.



Theorem: a^2 + b^2 = c^2

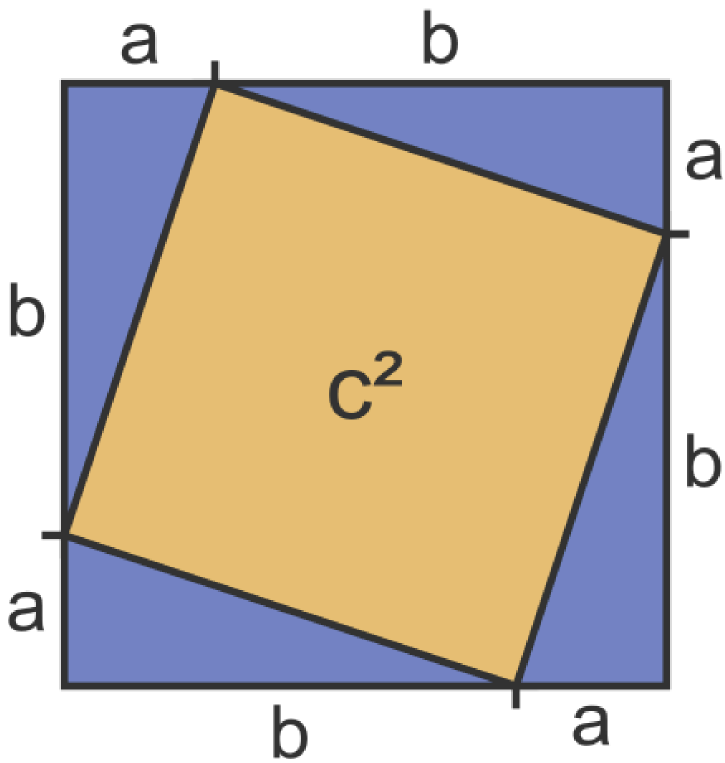
Let’s try a couple of examples using this theorem.

1. We have a triangle ABC with a right triangle at B. If AB = 4 and BC = 3, what is the length of AC?
   Ans: 5
2. We have a triangle ABC with a right triangle at B. If AB = 40 and AC = 41, what is the length of AB?
   Ans: 9

Some specific right triangles shown up often in problems, so memorizing a few can be helpful! For instance:

- 3, 4, 5
- 8, 15, 17
- 5, 12, 13

Keep in mind that if you find a Pythagorean triple that works, any multiple of that triple will also work.
Try it yourself! If a triangle with side lengths (3, 4, 5) works, then a triangle with side lengths (6, 8, 10) is guaranteed to work as well.
Now, why does the Pythagorean Theorem even work? There are several available proofs of the Pythagorean Theorem, but below is a fairly common one that uses algebra and areas.



So, let’s say that we have a square. Inside is another square with side length c. This means that the larger square is split up into four parts - one square with side length c and 4 equal rectangles.

Let’s name the sides of the rectangles a and b. Because the hypotenuse coincides with the side length of the inscribed square, we know the hypotenuse of the rectangles must be c.
This gives us 4 congruent rectangles with side lengths a, b, and c.

Next, let’s try to find the area of the larger square. Well, we know that the square has side length (a+b) so the area can be written as:
$(a+b)^2$

However, we see we can actually find the area another way - calculating the area of the four rectangles and the inscribed square separately, then adding them together.
The area of the four rectangles are:
$4 \times ab/2 = 2ab$
The area of the inscribed square is:
c^2
So adding them together, we get that the area is:
$2ab + c^2$

So now, we have found the area of the large square in two different ways. This means we can set them equal to each other.

$(a+b)^2 = 2ab + c^2$
$a^2 + 2ab + b^2 = 2ab + c^2$
There is 2ab on both sides, so they cancel out and we get…
$a^2 + b^2 = c^2$

Example problems using the Pythagorean Theorem:

1. There is a triangle ABD with C on BD so that AC is perpendicular to BD. If AB = 10, BC = 6, and CD = 15, what is the length of AD?

2. In a triangle ABC, AB = BC = 29 and AC = 42. What is the area of ABC?

3. The midpoints of the four sides of a rectangle are (-3,0), (2,0), (5,4), and (0,4). What is the area of this rectangle?

4. In rhombus ABCD, point P lies on segment AD so that BP is perpendicular to AD, AP = 3, and PD = 2. What is the area of ABCD?

ANSWERS:

1. 17
2. 420
3. 40
4. 20