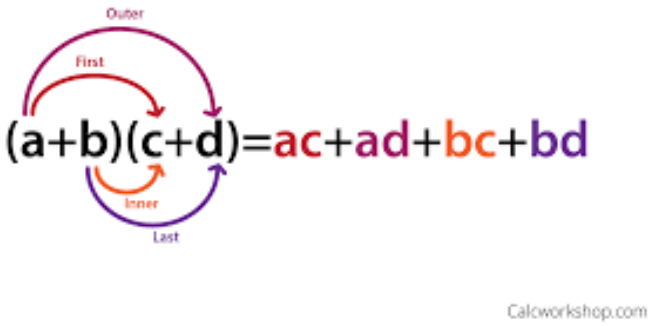
Math Circle

Expanding Polynomials

Polynomials are just sums of terms with variables raised to powers. The simplest building block is multiplying binomials.

FOIL stands for:

- **F**: First terms
- **O**: Outer terms
- **I**: Inner terms
- **L**: Last terms



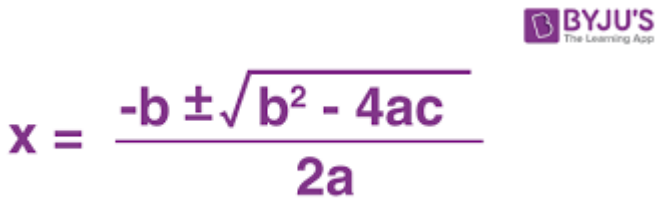
Example 1: (x + 3)(x + 5)

Example 2: (2x - 4)(x + 7)

Quadratic Formula

Every quadratic equation can be solved using the quadratic formula:

For ax^2 + bx + c = 0,



The part under the square root is the **discriminant** (b^2 - 4ac).

Example 1: Solve 2x^2 + 3x - 5 = 0.

Example 2: Solve x^2 - 6x + 10 = 0.

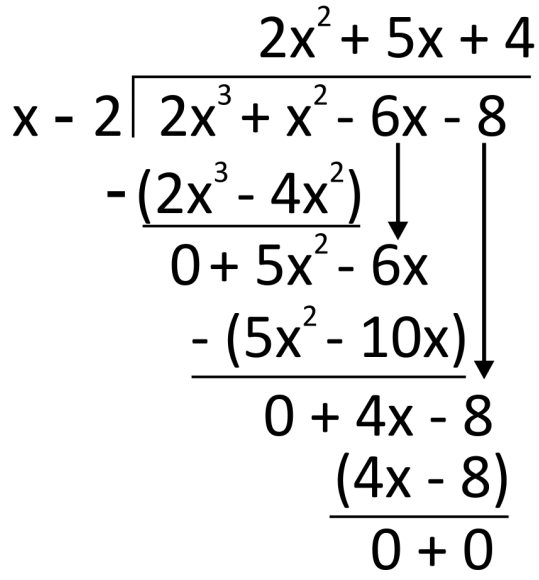
---

3. Polynomial Division

Polynomial long division works like normal long division with numbers.

Steps:

1. Divide the first term of the dividend by the first term of the divisor.
2. Multiply the divisor by that term.
3. Subtract this from the dividend.
4. Repeat with the new polynomial until the degree of the remainder is less than the degree of the divisor.



Example 1: Divide 2x^3 + 3x^2 - 5x + 6 by x - 2.

Example 2: Divide x^3 - 4x^2 + 5x - 2 by x - 1.

---

4. Remainder Theorem

The Remainder Theorem says:
If you divide f(x) by (x - r), the remainder is just f(r).

Example 1: f(x) = x^3 - 2x^2 + 3x - 7. Find remainder when dividing by (x - 2).

Example 2: f(x) = 2x^4 - 5x^2 + 4x + 1. Find remainder when dividing by (x + 3).

---

8 Challenge Problems

1. Expand (3x - 2)(x + 7) using FOIL.

2. Solve $3x^2 - 12x + 9 = 0$ using the quadratic formula.

3. Solve $5x^2 + 2x + 1 = 0$ using the quadratic formula.

4. Divide $3x^3 - 5x^2 + 6x - 4$ by $(x - 2)$.

5. Divide $x^4 + 2x^3 - x^2 + 5$ by $(x + 1)$.

6. Use the Remainder Theorem to find the remainder when $f(x) = x^3 + 4x^2 - x + 7$ is divided by $(x - 3)$.

7. Use the Remainder Theorem to find the remainder when $f(x) = 2x^4 - 3x^2 + 1$ is divided by $(x - 2)$.

8. A polynomial $f(x)$ leaves a remainder of 5 when divided by $(x - 1)$ and a remainder of -3 when divided by $(x + 2)$. Find $f(1)$ and $f(-2)$.