Math Circle 12/06

Warmup

1. Find the area of a triangle with base 10 and height 6.

2. Find the midpoint between (2, 6) and (8, 10).

3. A segment is divided in a 2:1 ratio. If the whole length is 15, what are the two parts?

Heron's Formula

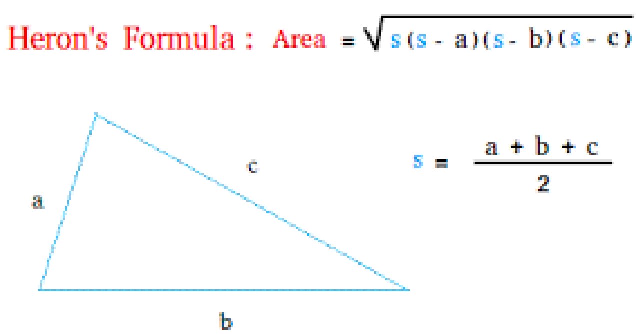
Heron's Formula allows you to find the area of a triangle if all three sides are known.

Steps:

1. Compute the semi-perimeter:
 $s = (a + b + c) / 2$

2. Compute the area:
 $\text{Area} = \text{sqrt}(s(s - a)(s - b)(s - c))$

Use this when height and angles are not given.



Example

Find the area of a triangle with sides 9, 10, and 17.

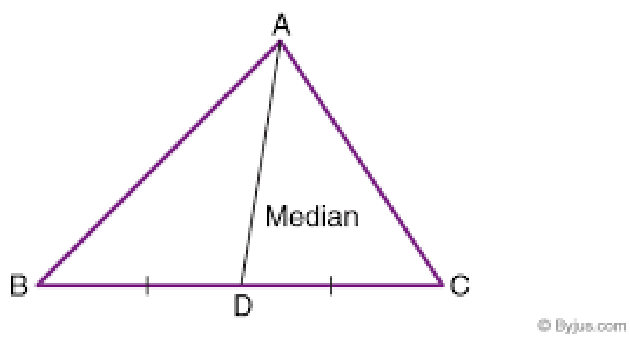
Step 1:
$s = (9 + 10 + 17) / 2 = 18$

Step 2:
$\text{Area} = \sqrt{18 \times 9 \times 8 \times 1}$
$\text{Area} = \sqrt{1296} = 36$

Practice Problems

A triangle has side lengths that are consecutive integers. Its area is 84. Find the perimeter.

Medians



A median is a segment drawn from a triangle’s vertex to the midpoint of the opposite side.

The three medians intersect at one point.
The medians are always divided in a 2:1 ratio at their intersection point.

Vertex to centroid is twice as long as centroid to midpoint.

Example

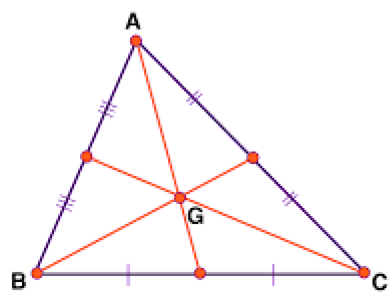
A median has length 24.

Break into two parts using the 2:1 ratio:
Vertex to centroid = 16
Centroid to midpoint = 8

The Centroid

The centroid is the point where all three medians meet.

In coordinate geometry, the centroid is found by averaging the x-coordinates and y-coordinates.



Formula:
Centroid = ((x1 + x2 + x3)/3 , (y1 + y2 + y3)/3)

Example (Centroid)

Triangle points:
(3, 3), (9, 3), (6, 12)

x-coordinate = 18 / 3 = 6
y-coordinate = 18 / 3 = 6

Centroid = (6, 6)

Practice Problems (Centroid)

1. Triangle ABC has vertices A(2, 4), B(10, 2), and C(x, y). The centroid lies on the line y = x and also on the segment connecting (4, 4) and (8, 8). Find (x, y).
2. The medians of triangle ABC intersect at G .A line through G parallel to BC meets AB at M and AC at N. If the area of triangle ABC is 144, find the area of triangle AMN.

Final Problems

1. The medians of triangle ABC intersect at G.
   A line through G parallel to side BC meets AB and AC at M and N.
   The area of triangle ABC is 180.
   Find the area of triangle AGN.
2. A triangle has area 120.
   A point inside the triangle is equally distant from all three vertices.
   Lines are drawn from that point to each vertex.
   The triangle is split into 3 regions.
   Find the area of each region.
3. A triangle's medians form a smaller triangle inside it.
   Find the ratio of the area of that triangle to the area of the original triangle.
4. In triangle ABC, point D lies on BC so that BD = 2 × DC.
   A line through D parallel to AB meets AC at E.
   If the area of triangle ABC is 90, find the area of triangle DEC
5. The centroid of triangle ABC is G.
   The line through G parallel to AC meets BC at D.
   If the area of triangle ABC is 72, find the area of triangle BGD.