

Math Circle

Combinations

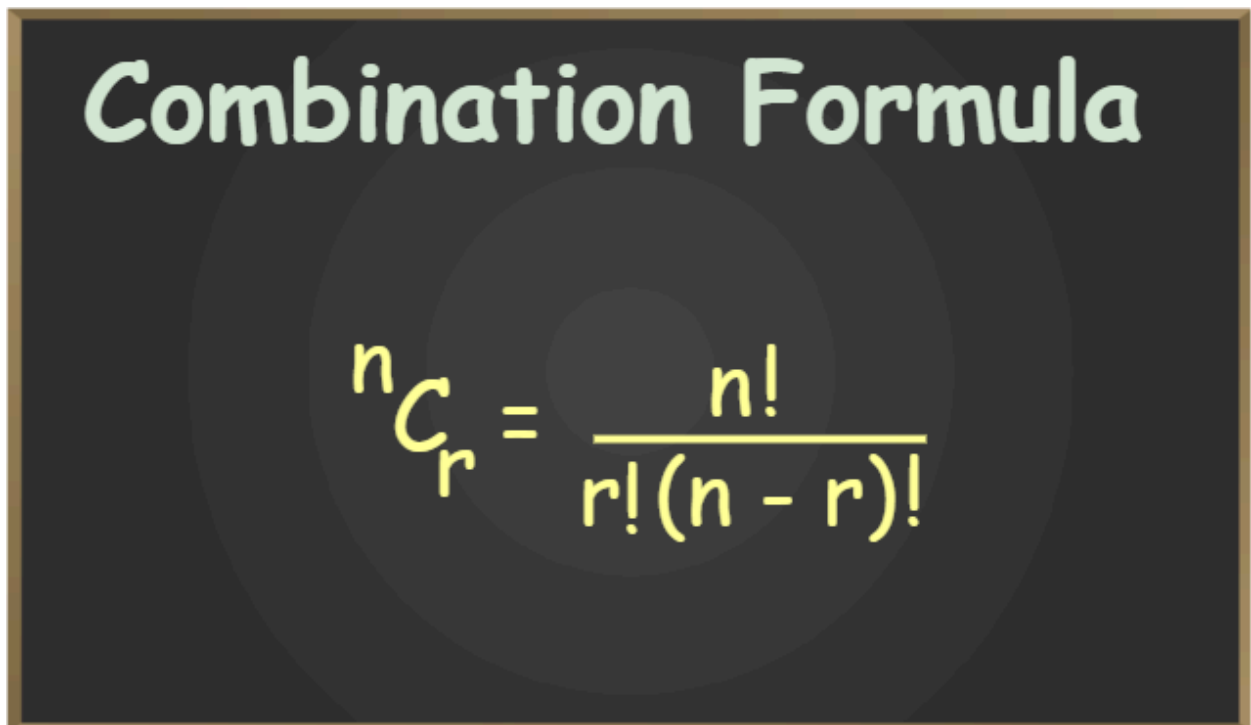
What Are Combinations?

A **combination** is a way of selecting items from a group **when order does not matter**.

- Example: Choosing 2 fruits from {apple, banana, cherry}
 - apple + banana is the same as banana + apple (only counted once)

If **order matters**, that's called a **permutation**, not a combination.

Combination Formula:



The image shows a chalkboard with a dark background and a light-colored border. At the top, the text "Combination Formula" is written in a light green, sans-serif font. Below this, the combination formula is written in a yellow, handwritten-style font. The formula is ${}^nC_r = \frac{n!}{r!(n-r)!}$. The variables n and r are subscripts, and the exclamation marks represent factorials.

Example:

How many ways can we choose 2 people from a group of 4?

$$\begin{aligned}C(4,2) &= \frac{4!}{2! \times (4-2)!} \\&= \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} \\&= 6\end{aligned}$$

There are **6 ways**.

Real-Life Examples of Combinations:

- Choosing toppings for a pizza
 - Selecting a committee from a club
 - Picking cards from a deck
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Warmup:

1. How many ways can you choose 2 pets from {cat, dog, bird}?
 2. How many ways can you choose 3 team members from 5 people?
 3. Is choosing a password where the order of characters matters a combination? (Yes or No?)
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Practice Problem 1:

How many ways can you choose 2 flavors from 5 ice cream flavors?

Explanation:

$$\begin{aligned}C(5,2) &= \frac{5!}{2! (5-2)!} \\&= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1}\end{aligned}$$

= 10

Answer: 10 ways

Practice Problem 2:

How many ways can you choose 3 books from 6 books?

Explanation:

$$\begin{aligned} C(6,3) &= 6! / 3! (6-3)! \\ &= 6 * 5 * 4 * 3 * 2 * 1 / 3 * 2 * 1 * 3 * 2 * 1 \\ &= 20 \end{aligned}$$

Answer: 20 ways

Practice Problem 3:

You have 8 different snacks. You choose 4 for a party. How many combinations?

Try By Yourself Problems:

1. $C(7,3)C(7, 3)C(7,3)$
2. $C(9,2)C(9, 2)C(9,2)$
3. $C(10,5)C(10, 5)C(10,5)$
4. How many ways to choose 2 colors from {red, blue, green, yellow}?
5. A committee needs 3 people from 8 members. How many ways?