Math Circle

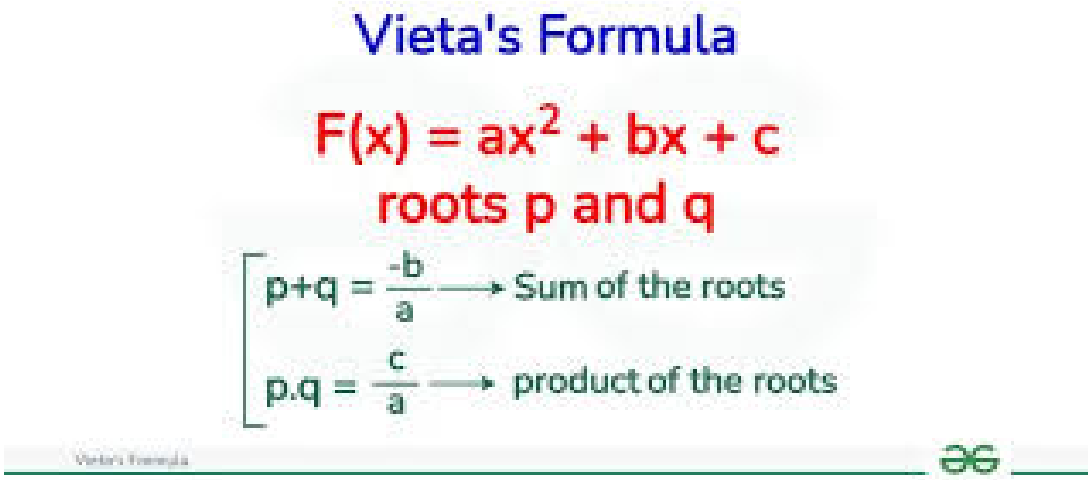
Sum and Product of Roots (Vieta's Formulas)

What are Vieta's Formulas?

Vieta's formulas connect the **coefficients** of a polynomial with the **sum and product of its roots**.

For a quadratic equation $\mathbf{ax^2 + bx + c = 0}$,
the two roots are $\mathbf{r_1}$ and $\mathbf{r_2}$, and:

- **Sum of roots:** $r_1 + r_2 = -b / a$
- **Product of roots:** $r_1 \times r_2 = c / a$



These relationships let us find the roots' sum and product **without solving the equation** directly.

Example:
$x^2 + 5x + 6 = 0$
Sum = -5 / 1 = -5
Product = 6 / 1 = 6
The roots (-2, -3) check out because (-2) + (-3) = -5 and (-2)(-3) = 6.

Warmup

1. Find the roots of $x^2 - 3x + 2 = 0$.
2. Find the sum and product of the roots of $x^2 + 7x + 12 = 0$.
3. For $2x^2 - 4x + 1 = 0$, what is the sum and product of the roots?

Practice Problem 1

Find the sum and product of the roots of $3x^2 + 2x - 1 = 0$.

Explanation:
Sum = $-b / a = -2 / 3$
Product = $c / a = -1 / 3$
Answer: Sum = -2/3, Product = -1/3

Practice Problem 2

The sum of two roots is 8 and their product is 15. Find the quadratic equation.

Explanation:
We know:
Sum = $8 \rightarrow -b / a = 8$
Product = $15 \rightarrow c / a = 15$
Let a = 1 for simplicity.
Equation $\rightarrow x^2 - 8x + 15 = 0$
Answer: $x^2 - 8x + 15 = 0$

Practice Problem 3

If one root of $x^2 - kx + 6 = 0$ is 2, find k.

Explanation:
Let the other root be r.
Product = $2r = 6 \rightarrow r = 3$
Sum = $2 + 3 = 5$

So, $-b/a = 5 \rightarrow k = 5$
Answer: $k = 5$

Practice Problem 4

The roots of a quadratic differ by 5 and have a product of 36. Find the equation.

Explanation:
Let roots be r and $r + 5$.
Product = $r(r + 5) = 36 \rightarrow r^2 + 5r - 36 = 0$
Solve: $r = (-5 \pm \sqrt{(25 + 144)}) / 2 = (-5 \pm 13) / 2 \rightarrow r = 4$ or -9
If $r = 4$, roots are 4 and 9.
Sum = 13, Product = 36
Equation $\rightarrow x^2 - 13x + 36 = 0$
Answer: $x^2 - 13x + 36 = 0$

Problems

1. Find the sum and product of the roots of $x^2 - 6x + 8 = 0$.
2. The roots of a quadratic are 2 and 5. Write the quadratic equation.
3. The sum of two roots is -4, and their product is 1. Find the equation.
4. If the roots of a quadratic are 3 and -7, find the equation.
5. The product of two roots is 10, and their sum is 9. Find the equation.
6. One root of $x^2 - px + 12 = 0$ is 3. Find p.
7. The roots of a quadratic are reciprocals of each other. If the equation is $4x^2 + bx + 9 = 0$, find b.
8. If the roots of a quadratic differ by 5 and their product is 36, find the equation.

Solutions

1. Sum = 6, Product = 8
2. $x^2 - 7x + 10 = 0$
3. $x^2 + 4x + 1 = 0$
4. $x^2 + 4x - 21 = 0$
5. $x^2 - 9x + 10 = 0$
6. $p = 7$
7. $b = 12$
8. $x^2 - 11x + 36 = 0$