Math Circle Basic Logarithms

Exponential Functions

Examples of exponential equations:

$2^3 = 8$

$10^2 = 100$

$5^{-1} = 1/5$

When we have exponential equations, we often need to find the exponent. For example:

- $2^x = 8 \rightarrow x = 3$
- $10^x = 100 \rightarrow x = 2$
- $5^x = 1/5 \rightarrow x = -1$

Logarithm Definition:

Definition: We say log_b(a) = x if $b^x = a$. This is read as "log base b of a equals x."

Examples of Logarithmic Equations:

- $\log_2(8) = 3$ because $2^3 = 8$
- $\log_{10}(100) = 2$ because $10^2 = 100$
- $\log_5(1/5) = -1$ because $5^{-1} = 1/5$
- $\log_3(1) = 0$ because $3^0 = 1$
- $\log_4(2) = 1/2$ because 4^(1/2) = √4 = 2

Converting Between Forms Example: Convert $3^4 = 81$ to logarithmic form

Answer: $\log_3(81) = 4$

Exercise 1 - Conversion Practice:

a) Convert $2^5 = 32$ to logarithmic form: _____

b) Convert $10^{-2} = 0.01$ to logarithmic form: _____

c) Convert $\log_7(49) = 2$ to exponential form: _____

d) Convert $\log_6(1/6) = -1$ to exponential form: _____

Exercise 2 - Evaluation Practice:

a) $\log_3(27) =$ _____

b) $\log_{10}(1000) =$ _____

c) $\log_5(125) =$ _____

d) $\log_2(1/8) =$ _____

Common and Natural Logarithms

- **Common logarithm:** $\log(x) = \log_{10}(x)$
- **Natural logarithm:** $\ln(x) = \log_e(x)$ where $e \approx 2.718$

Exercise 3 - Common and Natural Logs:

a) $\log(100) =$ _____

b) $\log(0.1) =$ _____

c) $\ln(e) =$ _____

d) ln(1) = ______

Logarithm Properties Product Rule: log_b(xy) = log_b(x) + log_b(y)

Quotient Rule: log_b(x/y) = log_b(x) - log_b(y)

Power Rule: log_b(x^n) = n·log_b(x)

Example: Simplify $\log_2(8 \times 4)$ $\log_2(8 \times 4) = \log_2(8) + \log_2(4) = 3 + 2 = 5$

Exercise 4 - Properties Practice:

a) $\log_3(9 \times 27)$ = ______

b) $\log_5(125/25)$ = ______

c) $\log_2(16^2)$ = ______

d) $\log_{10}(1000 \times 10)$ = ______

Solving Exponential Equations with Logarithms Example:

Solve $2^x = 10$

Take log of both sides:

x·log(2) = log(10)

So x = log(10)/log(2)

Exercise 5 - Exponential Equations:

a) $3^x = 27$, so x = ______

b) $5^x = 125$, so x = _____

c) $2^x = 64$, so x = _____

d) $10^x = 1000$, so x = _____

Practice Problems

1. Convert $4^3 = 64$ to logarithmic form.
2. Evaluate $\log_6(36)$.
3. Simplify $\log_2(32) + \log_2(2)$.
4. Solve $\log_3(x) = 2$.
5. Simplify $\log_5(25 \times 5)$.
6. Solve $3^x = 81$.
7. Evaluate $\log_{10}(100) - \log_{10}(10)$.
8. Solve $\log_2(x) + \log_2(4) = 5$.

Answer Key

1. Convert $4^3 = 64$ to logarithmic form: **$\log_4(64) = 3$**
2. Evaluate $\log_6(36)$: **2**
3. Simplify $\log_2(32) + \log_2(2)$: **6**
4. Solve $\log_3(x) = 2$: **$x = 9$**
5. Simplify $\log_5(25 \times 5)$: **3**
6. Solve $3^x = 81$: **$x = 4$**
7. Evaluate $\log_{10}(100) - \log_{10}(10)$: **1**
8. Solve $\log_2(x) + \log_2(4) = 5$: **$x = 8$**