Math Circle

Basic Modular Arithmetic

Integers

Examples of integers: -3, -1, 0, 1, 2, 15, 100

When we divide integers, we often get remainders. For example:

- 17 ÷ 5 = 3 remainder 2
- 23 ÷ 7 = 3 remainder 2
- 100 ÷ 12 = 8 remainder 4

Modular Arithmetic Definition

Definition: We say $a \equiv b \pmod{n}$ if a and b have the same remainder when divided by n.

This is read as "a is congruent to b modulo n."

Examples of Modular Congruences:

- $17 \equiv 2 \pmod{5}$ because $17 = 5\times3 + 2$ and $2 = 5\times0 + 2$
- $23 \equiv 2 \pmod{7}$ because $23 = 7\times3 + 2$ and $2 = 7\times0 + 2$
- $100 \equiv 4 \pmod{12}$ because $100 = 12\times8 + 4$ and $4 = 12\times0 + 4$
- $15 \equiv 0 \pmod{3}$ because $15 = 3\times5 + 0$ and $0 = 3\times0 + 0$
- $-8 \equiv 4 \pmod{6}$ because $-8 = 6\times(-2) + 4$ and $4 = 6\times0 + 4$

Addition in Modular Arithmetic

Rule: If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $a + c \equiv b + d \pmod{n}$

Example:

Find (17 + 23) mod 5

17 ≡ 2 (mod 5) and 23 ≡ 3 (mod 5)

So (17 + 23) ≡ (2 + 3) ≡ 5 ≡ 0 (mod 5)

Exercise 1 - Addition Practice:

a) (13 + 18) mod 7 =

b) (25 + 34) mod 8 =

c) (19 + 27) mod 6 =

d) (45 + 38) mod 9 =

General Rule: (a + b) mod n = ((a mod n) + (b mod n)) mod n

Multiplication in Modular Arithmetic

Rule: If a ≡ b (mod n) and c ≡ d (mod n), then a × c ≡ b × d (mod n)

Example:

Find (14 × 19) mod 6

14 ≡ 2 (mod 6) and 19 ≡ 1 (mod 6)

So (14 × 19) ≡ (2 × 1) ≡ 2 (mod 6)

Exercise 2 - Multiplication Practice:

a) (15 × 8) mod 4 =

b) (12 × 7) mod 5 =

c) (23 × 11) mod 9 =

d) (18 × 25) mod 7 =

General Rule: (a × b) mod n = ((a mod n) × (b mod n)) mod n

Clock Arithmetic Applications

Exercise 3 - Clock Problems:

a) If it's 8:00 AM now, what time will it be in 17 hours?

Answer: _____ (use mod 12)

b) If today is Tuesday (day 2), what day will it be in 100 days?

Answer: _____ (use mod 7, where Sunday=0, Monday=1, etc.)

Basic Divisibility Rules Using Mods

Exercise 4 - Divisibility Practice:

a) Is 2341 divisible by 3?

b) Is 4567 divisible by 9?

c) What is the remainder when 12345 is divided by 11?

Practice Problems

1. What is (123 + 89) mod 10?
2. What is (56 × 73) mod 9?
3. Is 37 ≡ 1 (mod 6)?
4. What is the smallest positive number x such that x ≡ 4 (mod 7) and x ≡ 2 (mod 5)?
5. What is (33 × 44 × 55) mod 5?
6. If it's 10:00 PM now, what time will it be in 50 hours? (Use mod 12)
7. If today is Thursday (day 4), what day will it be 365 days from now? (Use mod 7)
8. What is the remainder when 99999 is divided by 11?