

Math Circle – Factorials

Factorials

Definition:

We write $n!$ (read “n factorial”) to mean:

$$n! = n \times (n - 1) \times (n - 2) \times \dots \times 2 \times 1$$

Examples:

$$3! = 3 \times 2 \times 1 = 6$$

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Example Problem:

What would be $6!$?

Factorials in Counting

If there are n different objects, the number of ways to arrange them in order is $n!$.

Example:

How many ways can we arrange the letters in "CAT"?

Number of letters = 3 $\rightarrow 3! = 6$ ways.

Problem:

How many different ways can you rearrange DOG?

When Letters Repeat

If some letters are the same, divide by the factorial of the number of times each repeated letter appears:

$$\text{Number of arrangements} = \frac{n!}{p_1! \times p_2! \times \dots}$$

Where:

- n = total letters
- p_1 = number of times first repeated letter occurs
- p_2 = number of times second repeated letter occurs, etc.

Examples:

"MOON": $n = 4$, O repeats twice

Number of arrangements = $4! \div 2! = 24 \div 2 = 12$

"PEEP": $n = 4$, P repeats twice, E repeats twice

Number of arrangements = $4! \div (2! \times 2!) = 24 \div 4 = 6$

Exercise 1 – Factorial Practice

- a) $4! =$ _____
b) $5! =$ _____
c) $6! =$ _____
d) $0! =$ _____
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Exercise 2 – Arrangements with All Different Letters

Find the number of arrangements for:

- a) "DOG" → _____
b) "MATH" → _____
c) "SUN" → _____
d) "FISH" → _____
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Exercise 3 – Arrangements with Repeated Letters

Find the number of arrangements for:

- a) "FOOD" → _____
 - b) "TREE" → _____
 - c) "NOON" → _____
 - d) "LEVEL" → _____
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Exercise 4 – Mixed Practice

- a) A word has 3 letters, all different. How many arrangements? → _____
 - b) A word has 5 letters with 2 letters repeated twice. How many arrangements? → _____
 - c) How many arrangements for "BANANA"? (Hint: 6 letters, 3 As, 2 Ns) → _____
 - d) How many arrangements for "APPLE"? (Hint: 5 letters, 2 Ps) → _____
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Exercise 5 – Challenge Problems

- a) How many ways can you arrange the digits 1, 2, 3, 4, 5?
 - b) How many 4-digit numbers can you make with the digits 1, 2, 3, 4 without repeating?
 - c) How many different arrangements of the letters in "STATISTICS"? (Hint: 10 letters; S repeats 3 times, T repeats 3 times, I repeats 2 times, C repeats 1 time)
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Answer Key

Exercise 1:

- a) 24
- b) 120
- c) 720
- d) 1

Exercise 2:

- a) 6
- b) 24
- c) 6
- d) 24

Exercise 3:

- a) 12
- b) 12

- c) 6
- d) 30

Exercise 4:

- a) 6
- b) 30
- c) 60
- d) 60

Exercise 5:

- a) 120
- b) 24
- c) 50,400